Spatial predictive distribution for precipitation based on numerical weather predictions (NWP)

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TIES 2010

Outline

- Motivation
- Data
- Model
  - Quantile regression
- Results
- Summary
- Further work

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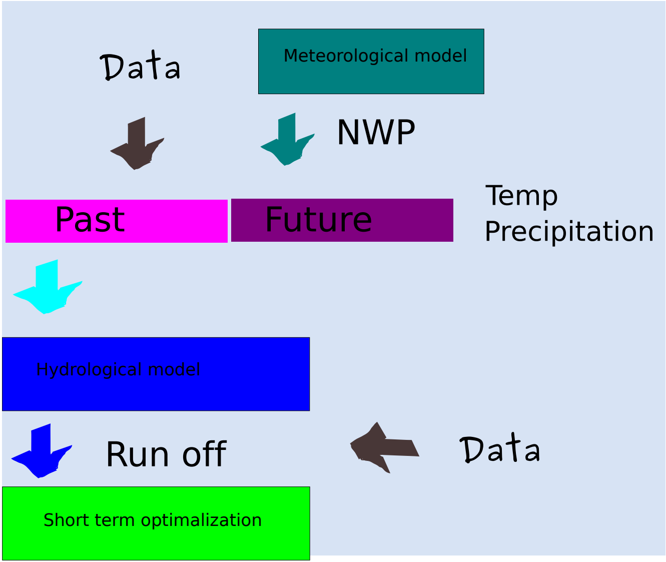
Probabilistic precipitation forecast using spatial quantile regression.

Motivation, hydro power production

How much water comes when? With uncertainty!

Motivation


Data
Meteorological model
NWP
Past
Future
Temp
Precipitation
Hydrological model
Run off
Data
Short term optimalization

Goal and method

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Find a spatial predictive distribution for precipitation based on NWP that is calibrated, sharp, fast to sample from and possible to interpolate to locations without observations.

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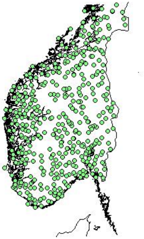
I'm a predictiveist

Care about the predictive distribution.

Data

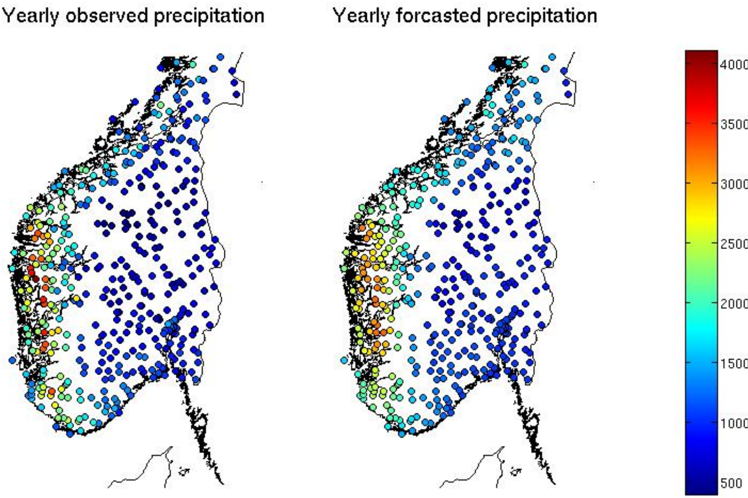
Southern Norway, 1. Jan. 2005 - 31. Aug. 2009.



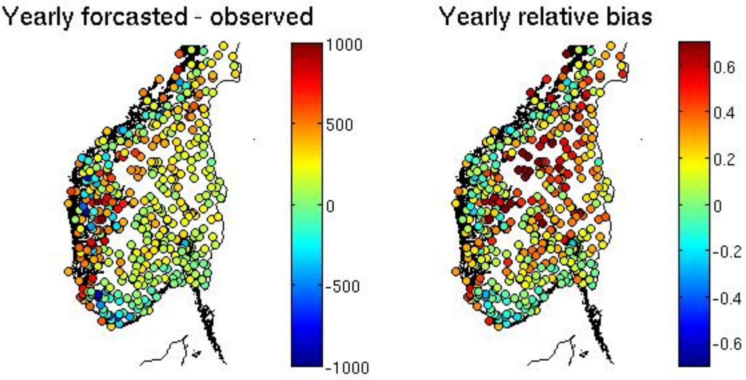
Observations: 436 sites, all with > 365 observations

Forecast:
- European Centre for Medium-Range Weather Forecasts (ECMWF)
- On 0.5×0.5 or 0.25×0.25 grid
  - Interpolate to observation sites in log/lat.
- $6, 12, 18, \ldots 240h$ forecast.
- Use 24 hour forecast, $30h - 6h$, for precipitation.

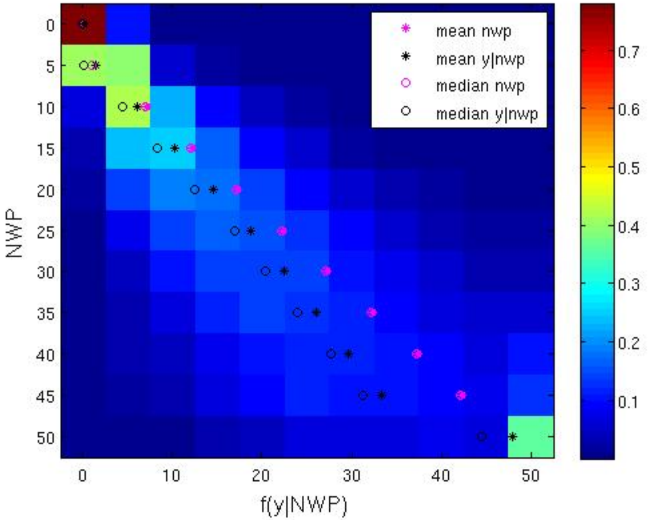
Exploratory analysis



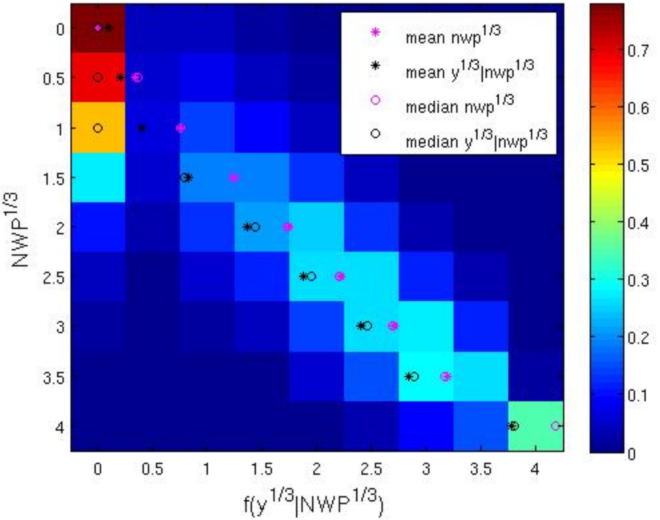
Exploratory analysis, bias



Exploratory analysis, Conditional NWP



Exploratory analysis, Conditional $NWP^{1/3}$



Background

- Berrocal, Raftery & Gneiting *Probabilistic quantitative precipitation field forecasting using a two-stage spatial model.* The annals of Applied Statistics, 2008.
- Bremnes *Probabilistic Forecast of Precipitation in Terms of Quantiles using NWP Model output.* Monthly Weather Review, 2004.
- Reich, Fuentes & Dunson *Bayesian spatial quantile regression.* Journal of American Statistical Socity, 2010.

Precipitation issues:

- Zero inflated.
- Observations ≥ 0

Model, one site

Precipitation issues:

- Zero inflated.
- Observations ≥ 0

$$f(y|x_{nwp}) = Pr(y = 0|x_{nwp}) f(y|x_{nwp}, y > 0)$$

$Pr(y = 0|x_{nwp})$: Precipitation occurrence.

$f(y|x_{nwp}, y > 0)$: Amount of precipitation.

Two stage model: Model these separately

Model, one site

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Two stage model: Model these separately

Precipitation occurrence: Probit model

Amount of precipitation: Quantile regression

Model precipitation occurrence, one site

Probit model: $Pr(y = 0|x_{nwp}) = probit(w)$,
Probit link: $W \geq 0 \Rightarrow y(s) > 0$ $W < 0 \Rightarrow y = 0$
Latent field of form: $W = \gamma_0 + \gamma_1 x_{nwp} + \epsilon$
Gaussian process: $\epsilon \sim N(0, 1)$

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W models

1. $W = \gamma_0 + \gamma_1 x_{nwp}^{1/3} + \epsilon$
2. $W = \gamma_0 + \gamma_1 x_{nwp}^{1/3} + \gamma_3 I(x_{nwp} = 0) + \epsilon$
3. $W = \gamma_0 + \gamma_1 \log(x_{nwp} + 0.1) + \epsilon$

Quantile regression

Linear regression

$$y|x = \beta_0 + \beta_1 x + \epsilon$$

$$E(y|x) = \beta_0 + \beta_1 x$$

Quantile regression

$$\tau = Pr(y \leq q_\tau | x)$$

$$q(\tau|x) = \beta_0(\tau) + \beta_1(\tau)x$$

Quantile regression

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Quantile regression

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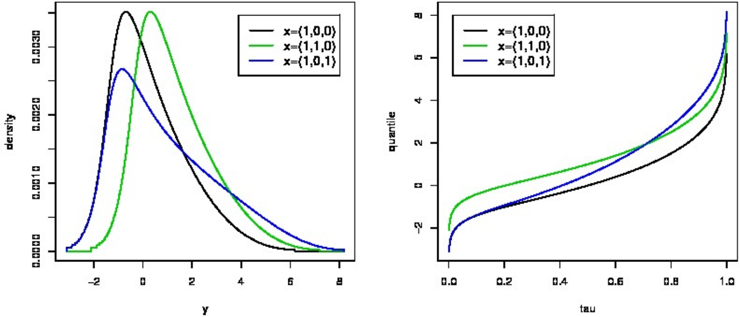
- A covariate can have different influence for different quantile.
- Do this for all τ: Model the inverse of the cumulative distribution function (cdf).

Example quantile regression

Example:

$$q(\tau) = (\tau + 1)\Phi^{-1}(\tau)x_1 + x_2 + 2\tau^2 x_3$$

- $\beta_1(\tau) = (\tau + 1)\Phi^{-1}(\tau)$ (Φ $N(0, 1)$ CDF)
- $\beta_2(\tau) = 1$
- $\beta_3(\tau) = 2\tau^2$



Classical quantile regression

The quantile regression estimator is:

$$\hat{\beta}(\tau_k) = argmin_{\beta} \sum_{y_i > x_i'\beta} \tau_k |y_i - x_i\beta| + \sum_{y_i < x_i'\beta} (\tau_k - 1)|y_i - x_i\beta|$$

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- Different quantiles analysed separately.

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- Different quantiles analysed separately.
- No guarantee that the quantiles don't overlap.

Quantile regression and Bernstein basis

Quantile regression:

$$q_\tau | x = \beta_0(\tau) + \beta_1(\tau)x$$

Use Bernstein basis polynomial for $\beta(\tau)$:

$$\beta(\tau) = \sum_{m=1}^{M} \alpha_m B_m(\tau)$$

where

$$B_m(\tau) = \binom{M}{m} \tau^m (1-\tau)^{M-m}$$

Useful properties:

- $\alpha_1 \leq \alpha_2 \cdots \leq \alpha_M \Rightarrow$ is a cdf.

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Useful properties:

- $\alpha_1 \leq \alpha_2 \cdots \leq \alpha_M \Rightarrow$ is a cdf.
- Quantiles do not cross.
- Can restrict $q_\tau \geq 0$ by setting $\alpha_1 = 0$

Model precipitation amount, $f(y|x_{nwp}, y > 0)$

For one location:
Model the τ quantiles:

$q(\tau)$ model

1. $q(\tau) = \beta_0(\tau) + \beta_1(\tau)x_{nwp} + \beta_2 x_{nwp}^{1/3}$.

with $\beta(\tau) = \sum_{m=1}^{M} \alpha_m B_m(\tau)$

- Set $\alpha_1 = 0$.
- Give truncated Gaussian priors to $\delta_i = \alpha_i - \alpha_{i-1}$.

Spatial model

Single site model:

Occurrence: $Pr(y = 0|x_{nwp}) = probit(w),\ W = \gamma_0 + \gamma_1 x_{nwp} + \epsilon$

Amount: $q_\tau = \sum_{m=1}^{M} \alpha_{0m} B_m(\tau) + \sum_{m=1}^{M} \alpha_{m1} B_m(\tau) x_{nwp}.$

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Separate spatial models for occurrence and amount.

Approximative inference, occurrence

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Ignore temporal and spatial dependence in the residuals.

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$$\hat{\beta}(s_i) = [\hat{\beta}_1(\tau_1, s_i), \ldots, \hat{\beta}_1(\tau_K, s_i), \hat{\beta}_2(\tau_1, s_i), \ldots, \hat{\beta}_2(\tau_K, s_i)]'$$

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Ignore temporal and spatial dependence in the quantile dependence.

Evaluation of predictive distribution

Training data: 1338 days

Test data: 365 days

Score function

Bineary response: Brier score

Continuous response: CRPS (continuous rank probability score)

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Score function

Bineary response: Brier score

Continuous response: CRPS (continuous rank probability score)

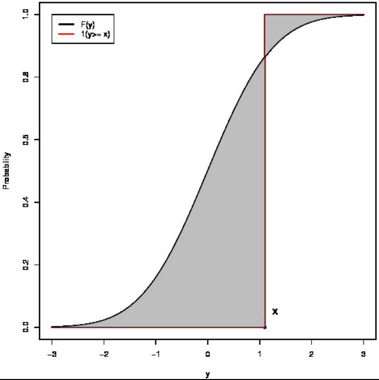
Brier score:

$$bs(f, o) = (f - y_{bin})^2$$

- $f = 1 - Pr(y = 0 | n_{nwp})$
- $y_{bin} = 0/1$ for no precipitation / precipitation.

$$crps(F - y) = \int_0^\infty (F(\psi) - I(y \leq \psi))^2 d\psi$$

- F : Cdf given forecast.
- $I()$: Index function



Results, precipitation occurrence

$$Pr(Y = 0|x_{nwp}) = probit(w)$$

W models

1. $W = \gamma_0 + \gamma_1 x_{nwp}^{1/3} + \epsilon$
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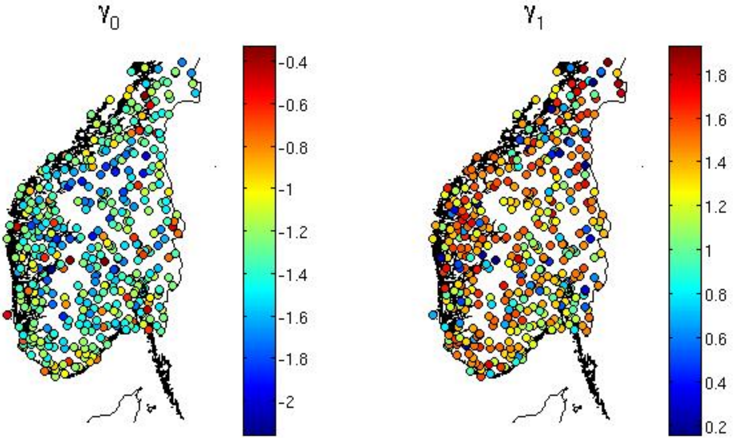
Mean Brier score:

Model	No space	Site wise	Spatial
1	0.1680	0.1659	**0.1656**
2	0.1680	0.1661	**0.1656**
3	0.1689	0.1663	0.1660
NWP			0.3003

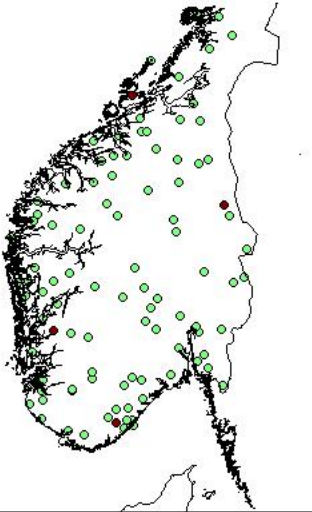
Results, precipitation occurrence



Results, precipitation amount

Use 100 sites.

Four sites: East, South, West, North



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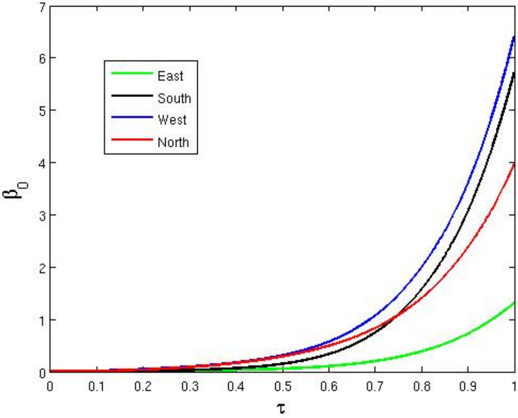

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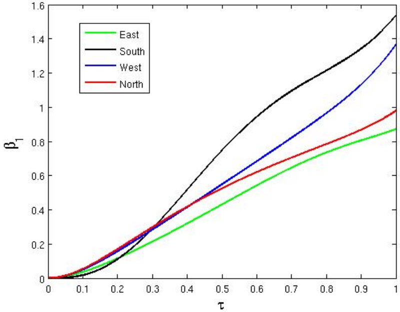

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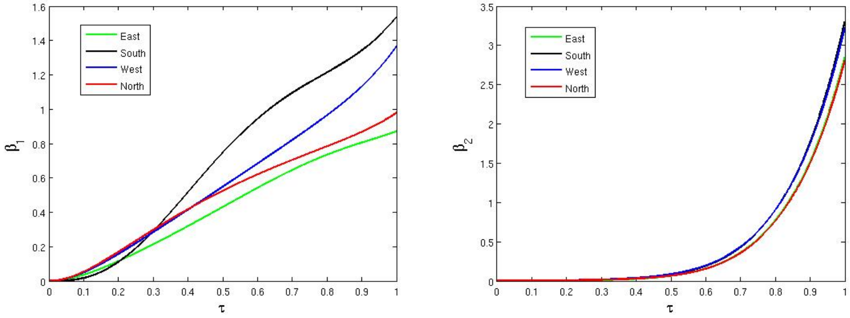

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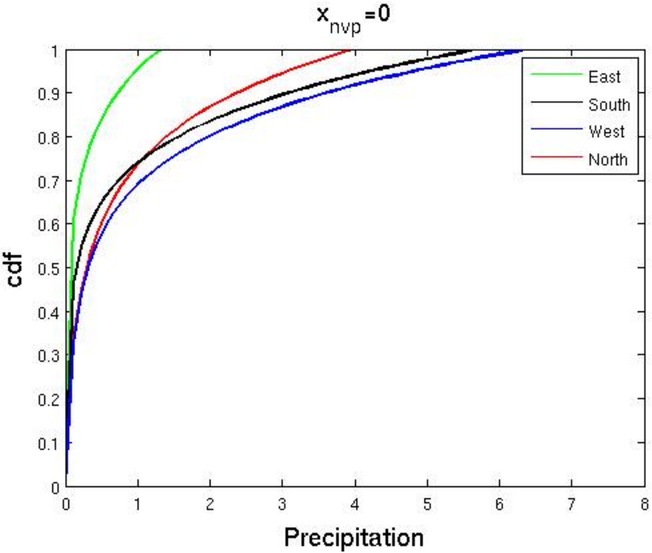

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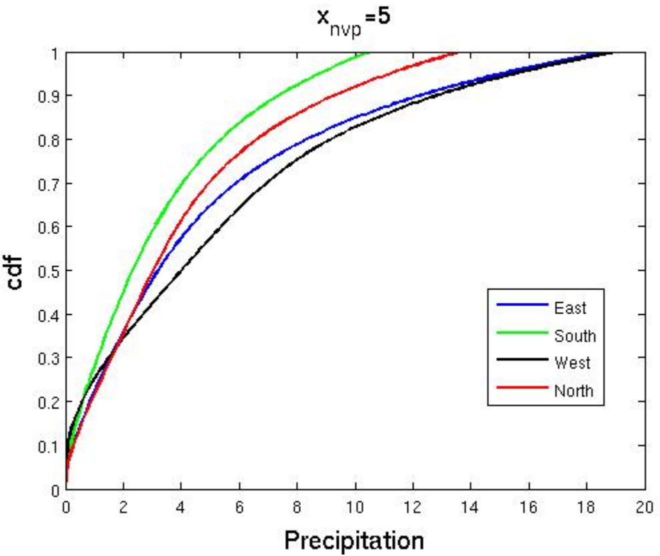

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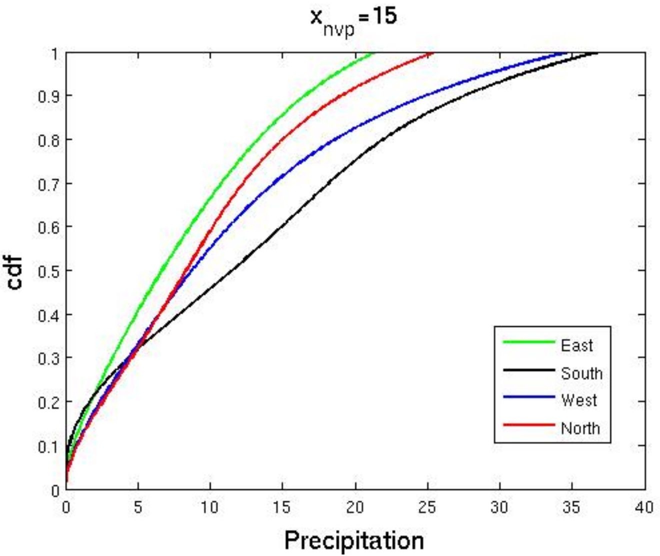
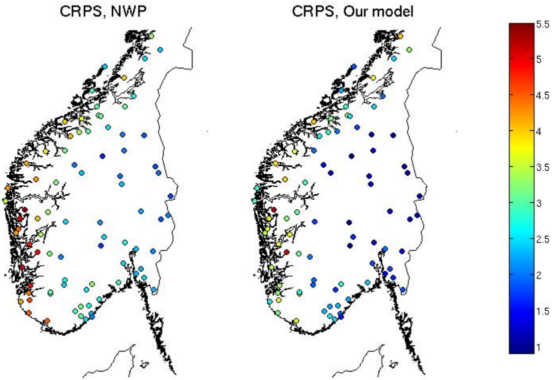


Results, full model



Mean CRPS:

- NWP: 3.05
- Our model: 2.43

Summary model and method

Model

- Seperate models for precipitation occurence and precipitation amount:

  Precipitation occurence: Spatial probit model
  Precipitation amount: Spatial quantile regression

- Spatial dependence in coefficiants.

Method

- Seperate inference for each stage of the model.
- Approximative inference for spatial model.

Review goal

Goal

Find a spatial predictive distribution for precipitation based on NWP that is calibrated, sharp, fast to sample from and possible to interpolate to locations without observations.

- Predictive distribution
- Calibrated and sharp
- Interpolate
- Fast to sample from.

Review goal

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- Predictive distribution OK
- Calibrated and sharp OK
- Interpolate OK
- Fast to sample from. OK

Extended model, more covariates

- Weather system proxy.
- Season.

Extended model, locations jointly

Spatial model:

- NWP: s
- Coefficients: s. Fixed in time.
- Residuals: s.

Occurence: $Pr(y(s) = 0|x_{nwp}(s)) = probit(w(s))$,
$W = \gamma_0(s) + \gamma_1(s)x_{nwp}(s) + \epsilon(s)$

Amount: $q_{\tau(s)}(s) = \sum_{m=1}^{M} \alpha_{0m}(s)B_m(\tau(s)) + \sum_{m=1}^{M} \alpha_{m1}(s)B_m(\tau(s))x_{nwp}(s)$.

Thank you!