Part 1: Geostatistical processes

Last week:
- How to model spatial continuous processes.
  - Hierarchical models
  - Gaussian Random Field (GRF)
  - Covariance functions (and variograms)

This week:
- Predictors for $Y|Z$ (assume Z known) (today)
- Estimation of covariance function (parameters θ) / variogram.
- Simulations + other topics.

2.1.4 Optimal Spatial Prediction

Today: Want to find predictor for process, i.e. $\hat{Y}|Z,\theta$.
Two approaches

Model based geostatistics:
- Make assumptions about model.
- Find posterior, $[Y|Z,]$.
- Optimal predictor under square loss is posterior mean, $E([Y|Z])$

Kriging:
- Make assumptions about the predictor
- Find optimal linear predictor

- We assume that (some of) θ is known.
- Main focus on model based geostatistics (more kriging in exercises)
- When do these approaches meet?

Hierachical statistical models (HM) for Mountain

Temperature:

Data model: $[Z|Y,\theta]$ Observations given true temperature:
Independent $Z(s_i) = N(Y(s_i), \sigma_\epsilon^2)$

Process model: $[Y|\theta]$ Distribution for temperature.
$Y(s) = \beta_0 + \beta_1 h(s) + \delta(\theta)$ where $\delta \sim GRF$ with
$E(\delta) = 0$ and covariance function $C_Y(s_1, s_2)$

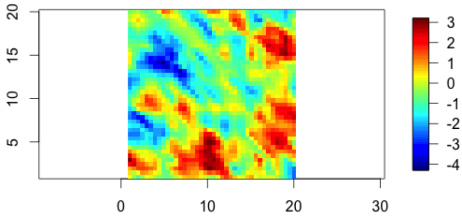
Parameter model: $[\theta]$ Prior for parameters. $[\theta] = [\sigma_\epsilon^2][\beta][C_Y()]$.

- $h(s)$ is elevation (meters above see level)
- Can write as matrices: $Y(s) = X\beta$ with $X = [1, h(s)]^T$ and $\beta = [\beta_0, \beta_1]^T$

Gaussian Random Field

Gaussian Random Field (GRF)

The random field $Y(s)$, $s \in D$ (f.ex. in R^2) is a Gaussian Random field if, for any n, and any set of locations $s_1, s_2, \ldots, s_n$, all finite collections $(Y(s_1), Y(s_2), \ldots Y(s_n))$ are multivariate Normal distributed.



Covariance function: $Cov(Y(s_1), Y(s_2)) = C_Y(s_1, s_2)$

Multivariate Normal distribution

Multivariate Normal(MVN) density

$Y = (Y_1, Y_2, \dots, Y_n)$ is MVN with expected value μ and covarance Σ, $Y \sim MVN(\mu, \Sigma)$ if

$$f(y) = \frac{1}{\sqrt{((2\pi)^n}} \exp(-\frac{1}{2}(y-\mu)^T \Sigma^{-1} (t-\mu))$$

Conditional MVN

Let $Y = (Y_1, Y_2)^T$, $\mu = (\mu_1, \mu_2)^T$ and

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}.$$

Then the $[Y_1 | Y_2 = a] \sim MVN(\mu_{1|2}, \Sigma_{1|2})$ with

- $\mu_{1|2} = \mu_1 - \Sigma_{12}\Sigma_{22}^{-1}(a - \mu_2)$ and
- $\Sigma_{1|2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

Example: Temperature difference (pg 134-135)

$$Y(\mathbf{s}) = \mathbf{x}(\mathbf{s})'\boldsymbol{\beta} + \delta(\mathbf{s}), \qquad \mathbf{s} \in D_s\,,$$

where for $\mathbf{s} = (s_1, s_2)'$, $\mathbf{x}(\mathbf{s}) = (1, s_1, s_2, s_2^2, s_2^3)'$, and $\boldsymbol{\beta} \equiv (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)'$

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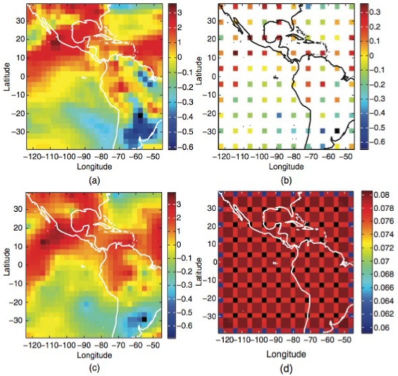

Figure 4.6 (a) Same plot as Figure 4.5a: Temperature change (1990s minus 1980s); the 28 × 28 values represent "the truth." (b) The 10 × 10 observations are obtained by subsampling "the truth" and adding mean-zero Gaussian noise. (c) Simple kriging predictor obtained from the missing and noisy data in (b). (d) Kriging standard error corresponding to simple kriging; the pattern is expected due to regular sampling in (b).