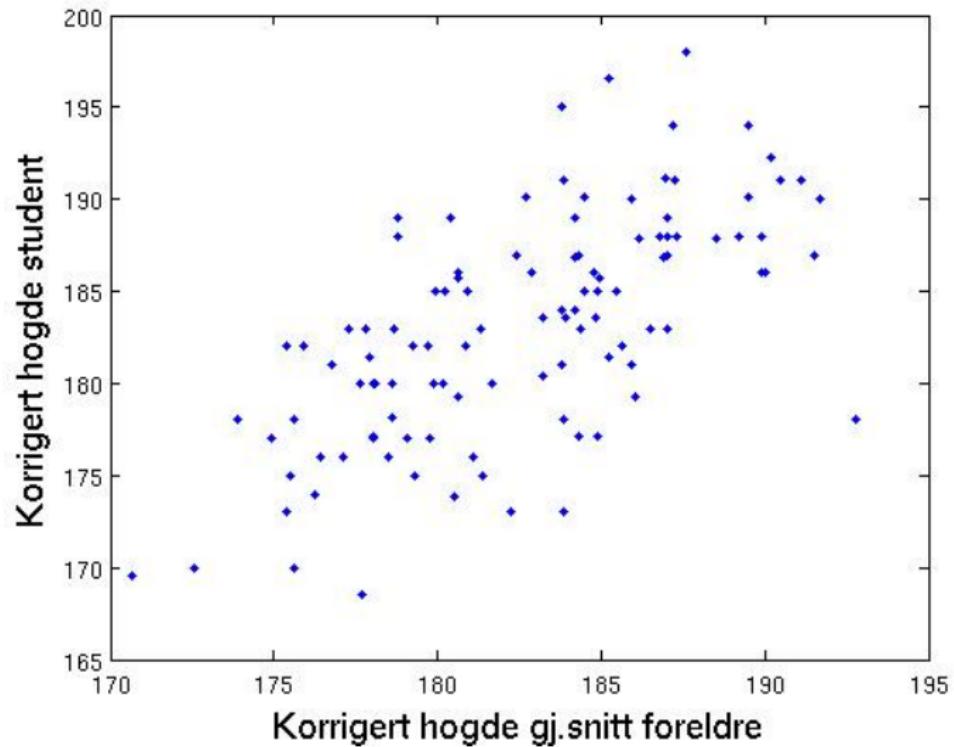


## Våre høgdedata



# Enkel lineær regresjon

$$Y_i = \alpha + \beta x_i + \epsilon$$

- ▶  $Y$ : Respons (stok. var)
- ▶  $x_i$ : Forklарingsvariabel (kjent, tal)
- ▶  $\alpha$  og  $\beta$ : Regresjonsparameter (param, tal, ukjent)
- ▶  $\epsilon$ : 'støyen', 'feilen', stok.var
  - ▶  $E(\epsilon) = 0$ ,  $Var(\epsilon) = \sigma_\epsilon^2$
  - ▶  $\sigma_\epsilon^2$  (param., tal, ukjent)

Dersom  $\epsilon \sim N(0, \sigma_\epsilon^2)$

$$Y_i \sim N(\alpha + \beta x_i, \sigma_\epsilon^2)$$

# Regresjonsparametra

Må estimere  $\alpha$ ,  $\beta$ ,  $\sigma_\epsilon^2$  frå data.

Data:  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ .

## Estimatorar

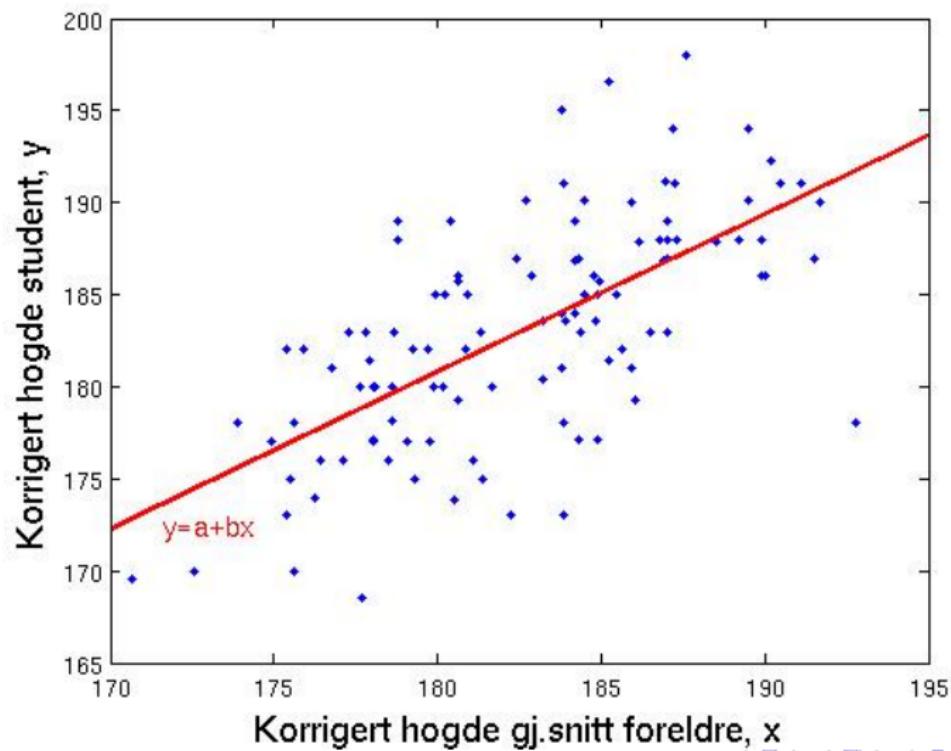
- ▶  $A = \bar{Y} - B\bar{x}$
- ▶  $B = \frac{\sum_{i=1}^n (x_i - \bar{x}) Y_i}{\sum_{i=1}^n (x_i - \bar{x})^2}$
- ▶  $S_\epsilon^2 = \frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n - 2}$
- ▶ Der  $\hat{Y}_i = A + Bx_i$

## Estimat

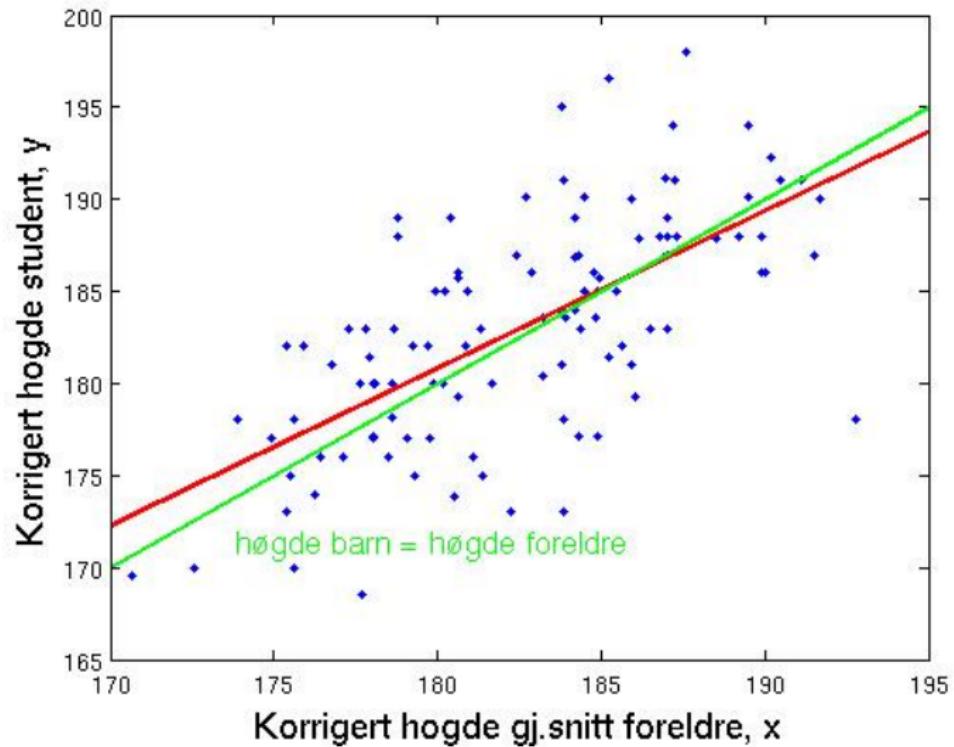
- ▶ Stok.var → data
- ▶  $a$ ,  $b$ ,  $s_\epsilon^2$ ,  $\hat{y}$

## Regresjonslinje (95% KI), våre data $n = 110$

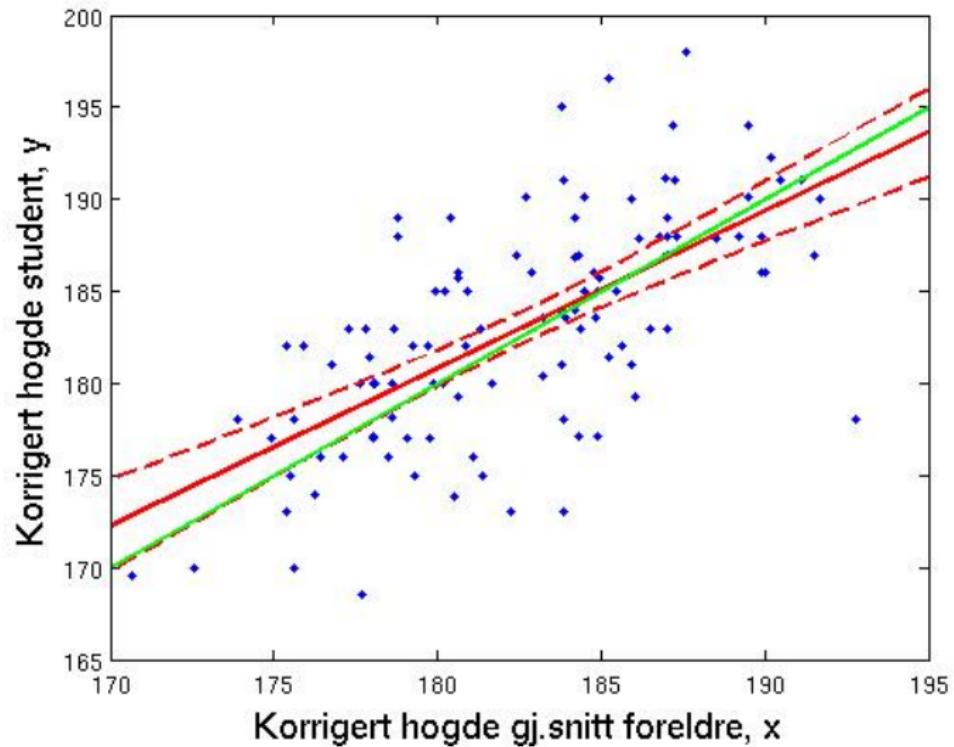
Estimat:  $a = 27.2$  ( $-6.6451, 60.9989$ ),  $b = 0.85$  ( $0.6684, 1.0387$ )  
og  $s^2 = 4.64^2$ .



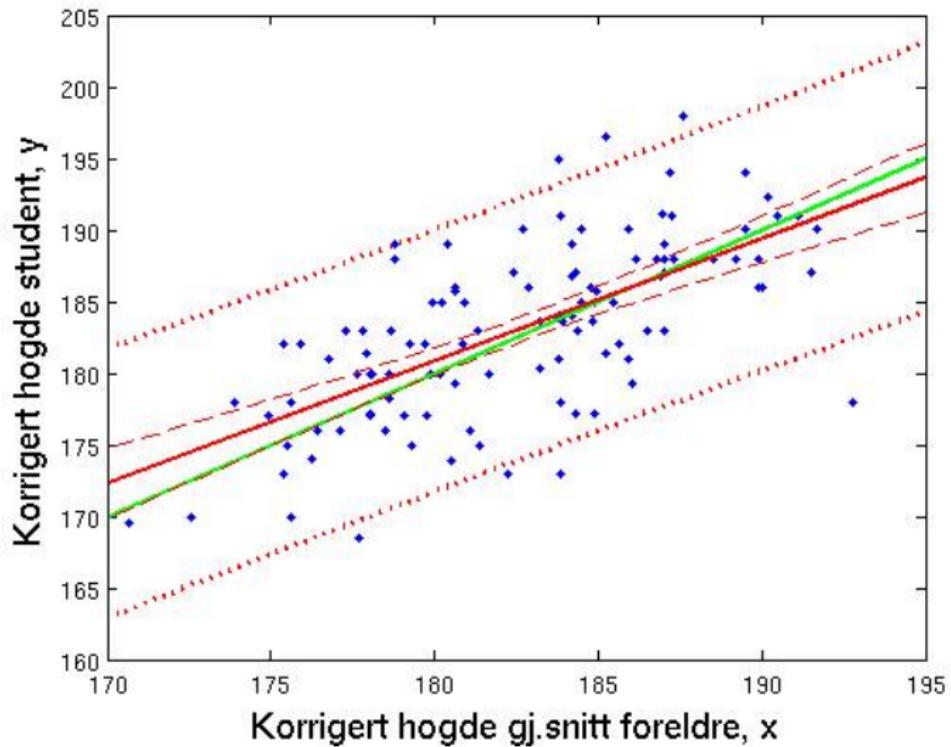
# Høgde barn = høgde foreldre?



## 95% konfidensintervall



## 95% prediksjonsintervall

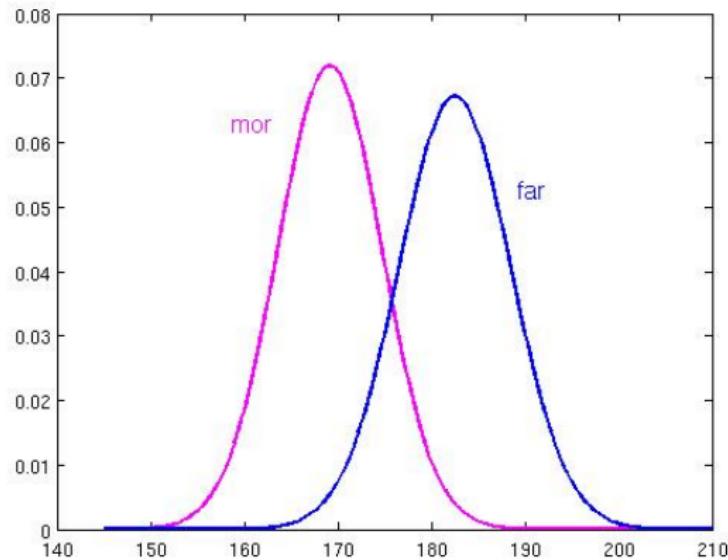


## Kap. 11.12 Korrelasjon

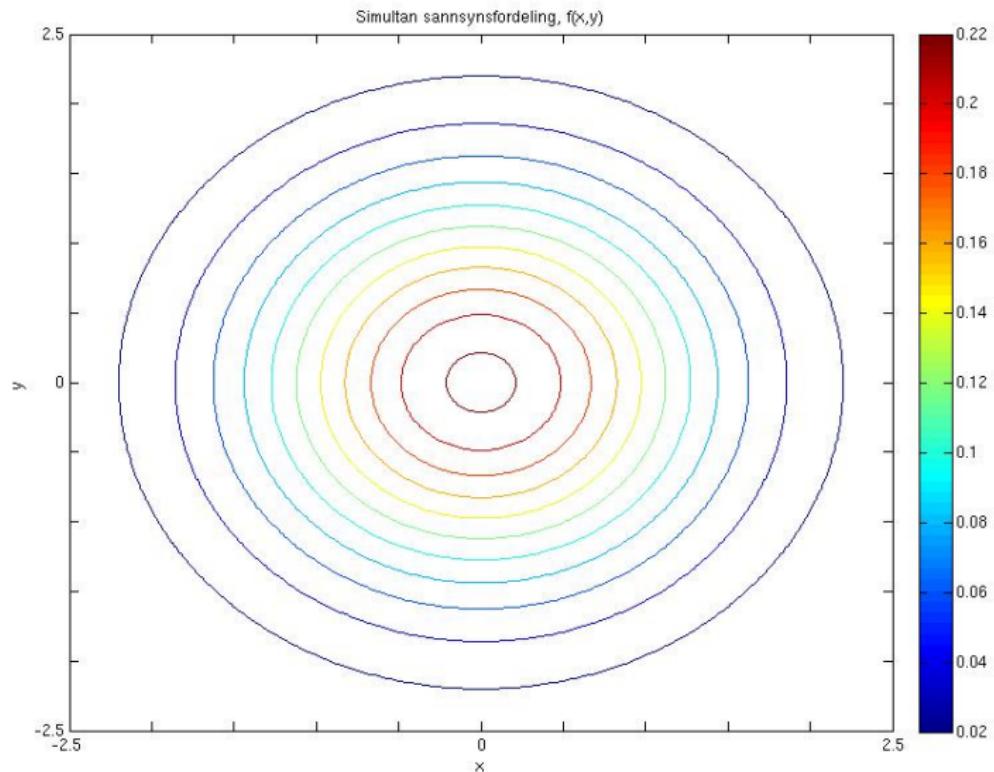
Skal binde saman:

- ▶ Kap. 3.4 Simultan sannsynsfordeling
  - ▶ To stok.var.,  $X$  og  $Y$ ,  $f(x, y)$
- ▶ Kap. 4.2 Varians og kovarians av stokastiske variable
  - ▶ Kovarians  $\sigma_{XY} = E((X - \mu_X)(Y - \mu_Y))$
  - ▶ Korrelasjon  $\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ ,  $-1 < \rho < 1$
- ▶ Kap. 11.2 Enkel lineær regresjon
  - ▶ Kjenner forklaringsvariabelen  $x$ .  $Y = \alpha + \beta x + \epsilon$ ,  $\epsilon \sim N(0, \sigma_\epsilon^2)$ .
- ▶ Kap. 11.5 ....  $R^2$ : Tilpassingskoeffisienten.
  - ▶ 
$$R^2 = 1 - \frac{\sum_{i=1}^n e_i^2}{\sum_{i=1}^n (y_i - \bar{y})^2}.$$
 $e_i$ : Residual

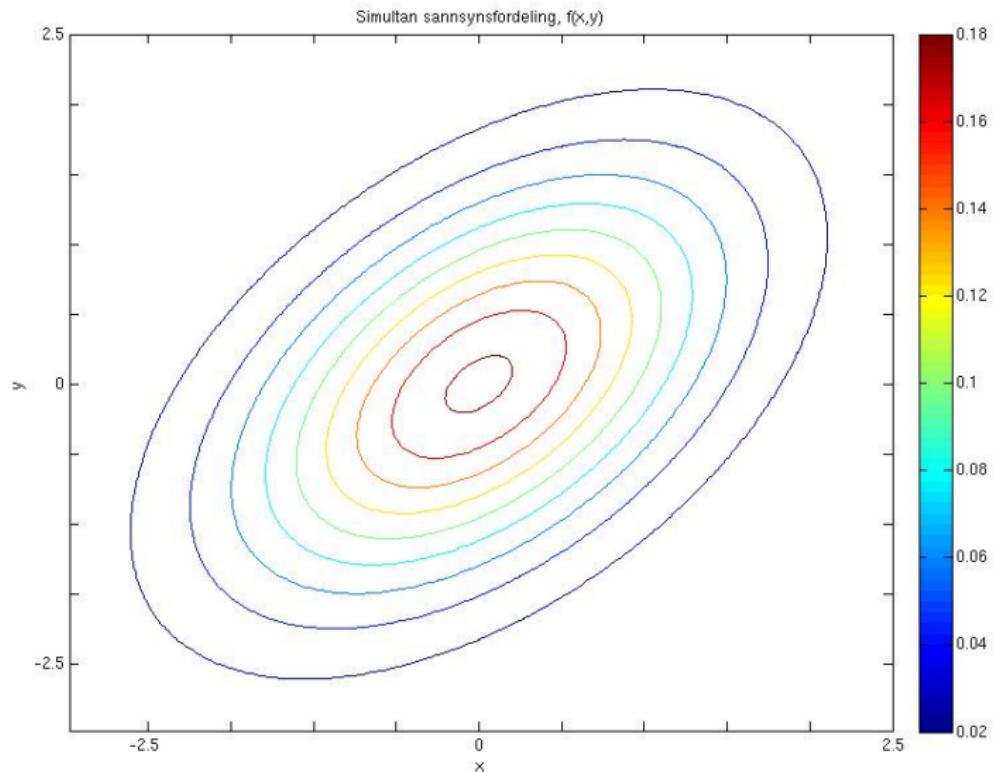
## Mor - far marginale sanns.fordeling og data



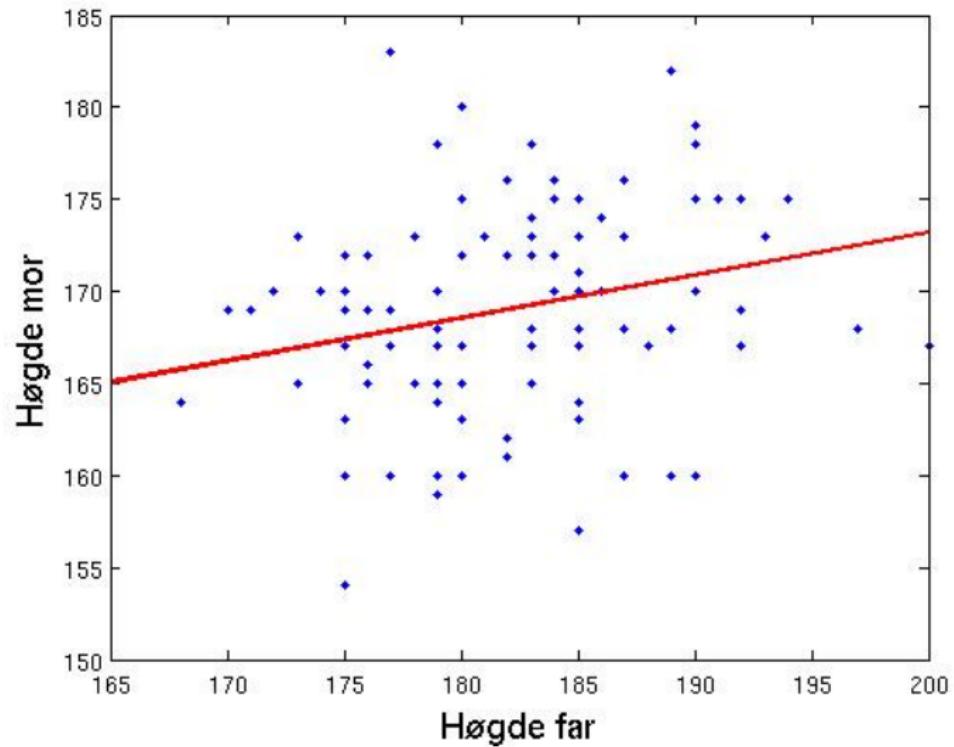
# Simultanfordeling 1, marginale N(0,1)



# Simultanfordeling 2, marginale N(0,1)



## Lineær modell høgde far høgde mor



## Simultanfordeling høgde far høgde mor

