# Introduction Chp 3

### Linear Model

- **1** Response:  $Y_i \sim N(\mu_i, \sigma^2)$ ,  $Y_i$ s independent
- 2 Linear relationship:  $E(Y_i) = \mu_i = x_i^T \beta_i$

We extend this class of models by:

- Response from exponential family of distributions.
  - ② Non-linear link:  $g(\mu_i) = x_i^T \beta$

# Exponential family

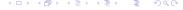
#### Definition

 $f(y;\theta)$  belongs to the exponential family if

$$f(y; \theta) = \exp[a(y)b(\theta)) + c(\theta) + d(y)]$$

### Examples:

- Normal
- Binomial
- Poisson
- Chi-square
- Gamma
- Beta



## Historical Linguistics

- Inspired by Ch 3.5.2.
- Interested in languages that descend from the same historical languages.
  - Norwegian and Swedish from Norse.
  - Modern French and Spanish from Latin.
- Languages that are separated by time t.
- Probability that a particular meaning has cognate words,  $\exp(-\lambda t)$ .
- Data: A linguist (Clue) judges if N different meanings are cognate:

Meaning	Norwegian	Swedish	Cognate
Laugh	Le	Skratta	No
House	Hus	Hus	Yes
Similar data for Spanish and French.			

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Explanatory variables and parameters

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  - $Y_i \sim bin(1, p_i)$  with  $p_i = exp(-\lambda t_i) = \theta_i$
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3 Explanatory variables and parameters

Parameter:  $\beta = \lambda$ 

Explanatory variable:  $t_i$ , time since separation.

 $X = [t_{ns}, t_{ns}, \dots t_{ns}, t_{sf}, \dots, t_{sp}]$ . t-s are assumed known.

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  - $\mu_i = E(Y_i) = p_i = \exp(-\lambda t_i)$
  - $g(\mu_i) = log(\mu_i)$
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#### Ideas extended model:

- Categories of meanings:
  - ★ Feelings
  - ★ Body parts
  - Mathematical terms
- Number of syllable.

