Exponential family

Definision

$f(y;\theta)$ belongs to the exponential family if

$$f(y;\theta) = \exp[a(y)b(\theta)) + c(\theta) + d(y)]$$

Examples:

- Normal
- Binominal
- Poisson
- Chi-sqiare
- Gamma
- Beta

Historical Linguistics

- Inspired by Ch 3.5.2.
- Interested in languages that descend from the same historical languages.
  - Norwegian and Swedish from Norse.
  - Modern French and Spanish from Latin.
- Languages that are separated by time t.
- Probability that a particular meaning has cognate words, $\exp(-\lambda t)$.
- Data: A linguist (*Clue*) judges if N different meanings are cognate:

Meaning	**Norwegian**	**Swedish**	**Cognate**
Laugh	Le	Skratta	No
House	Hus	Hus	Yes

Similar data for Spanish and French.

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2. Link function, $g(\mu_i) = x_i^T \beta$
3. Explanatory variables and parameters

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   Explanatory variable: t_i, time since separation.

   $X = [t_{ns}, t_{ns}, \ldots t_{ns}, t_{sf}, \ldots, t_{sp}]$. t-s are assumed known.

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   Ideas extended model:
   - Categories of meanings:
     - Feelings
     - Body parts
     - Mathematical terms
   - Number of syllable.